ACSIJ Advances in Computer Science: an International Journal, Vol. 2, Issue 3, No.4, July 2013
ISSN : 2322-5157
www.ACSIJ.org

A New Heuristic Constructing Minimal Steiner Trees inside Simple Polygons

Alireza Khosravinejad [1], Alireza Bagheri [2]

[1] **Department of Computer Engineering, Qazvin Branch, Islamic Azad University**
Qazvin, Iran
Betacoinfo@yahoo.com

[2] **Department of Computer Engineering and Information Technology Amirkabir University of Technology**
Tehran, Iran
ar_bagheri@aut.ac.ir

Abstract

The Steiner tree problem has numerous applications in urban transportation network, design of electronic integrated circuits, and computer network routing. This problem aims at finding a minimum Steiner tree in the Euclidean space, the distance between each two edges of which has the least cost. This problem is considered as an NP-hard one. Assuming the simple polygon P with m vertices and n terminals, the purpose of the minimum Steiner tree in the Euclidean space is to connect the n terminals existing in p. In the proposed algorithm, obtaining optimal responses will be sought by turning this problem into the Steiner tree problem on a graph.

Keywords: *Euclidean Steiner Minimal Tree, Delaunay triangulation, geodesic convex hull.*

1. Introduction

The Steiner problem is applied in several scientific and business applications, such as computer networks routing, electronic integrated circuits, petroleum shaft and post networks. The computational features of this problem make it an important research subject in computational geometry. Having some points in the Euclidean plane, the shortest path for connecting these points leads to a tree which is called Euclidean Steiner Minimal Tree (ESMT). The Euclidean Steiner minimal tree problem is considered as a NP-hard problem [1]. ESMT may contain some nodes that are not in the set of the given nodes that are known as Steiner nodes and we call the given nodes as terminals. ESMT in a plane without any obstacles consists of unions of ESMTs with few terminals. It is unusual to encounter ESMTs with 6 or more terminals in literature (which is a serious constraint and our approach tends to be free of it) [2] and ESMTs connecting subsets of up to 4 terminals have proved to yield good quality solutions for the obstacle-free cases [3, 4]. Optimal solution algorithms for the Euclidean Steiner problem have been presented by Boyce and Seery [5], Cockayne and Schiller [6], Winter [7] and Cockayne and Hewgill [8]. These algorithms work by examining topologies (a topology being a set of vertices and their associated edges) corresponding to full Steiner trees. Chang [9] presented an early heuristic algorithm based upon inserting vertices into the Minimum Spanning Tree (MST) in order to reduce the cost of the tree. This is a natural approach and has been used in many algorithms, for example Korhonen [10] and Smith and Leibman [11]. Smith, Lee and Leibman [4] presented an algorithm based upon Voronoi diagrams and Delaunay triangulations. Lundy [12] presented an algorithm based upon simulated annealing. Beasly in [3] have proposed a heuristic based upon finding optimal Steiner solutions for connected subgraphs of the minimum spanning tree of the entire vertex set. In this paper, we propose a new algorithm based upon straight skeleton of simple polygon to solve the problem in a Euclidean plan for any number of terminals. Finally, we compare our results in Euclidean plane with data and results presented in [13]. The paper is organized as follows: In Section 2 is dedicated to some basic definitions. In section 3, construction of the Steiner tree for three points is presented. In section 4, the proposed algorithm is explained. In section 5, the calculation results are presented. The final section is the conclusion.

2. Basic definitions

A polygon P is simple if it is not self-intersecting and its interior $i(p)$ is not empty and connected. A point P is said to be in P if $p \in i(p) \cup p$ .A vertex v on P is convex if its interior angle is less than 180°.

Otherwise, it is reflex. A reflex vertex is said to be wide if its interior angle is at least $240°$. Clockwise successor and predecessor vertices of a vertex v are denoted by v^+ and v^-, respectively. In order to simplify some proofs, it is assumed that v^-v and vv^+ are not colinear for any $v \in p$. A simple polygon is called a $c-\text{kite}$ iff precisely c of its vertices is convex. Boundaries of a $c-\text{kite}$ P between two consecutive convex vertices are referred to as sides of P. A polygon P is weakly-simple if it is not self-intersecting. In particular, a weakly-simple polygon can have empty or disconnected interior.

The shortest path between two points u and v in a polygon P will be denoted by $P(u,v)$. $P(u,v)$ is a unique polygonal chain and its interior vertices are reflex vertices of P. A line L is said to be an interior tangent of a c-kite P at a touch vertex $v \in P$ iff one of the following cases occurs.

- v is a reflex vertex, and edges v^-v and vv^+ are on the same side of L.
- v is a convex vertex, and edges v^-v and vv^+ are on the opposite sides of L.
- v^-v overlaps with L .

An interior tangent L with a touch-point v is oriented in such a way that the edge vv^+ is on its left. Two interior tangents of a c-kite P are distinct if they have different slopes or different touch vertices. Consider a reflex vertex v of a c-kite P . let q_v^- and q_v^+ denote the convex vertex that is reached form v by moving counterclockwise and (respectively clockwise) on P. Let sv denote an edge in p overlapping with an interior tangent of v . Only one of the vertices v^- and v^+ is visible form s. Let q_v^s denote the convex vertex that can be reached form v by moving counterclockwise on P in v^- is invisible from s, and by moving clockwise if v^+ is invisible from s. if v is convex, let $q_v^s = v$.

An ESMT inside a simple polygon cannot have vertices of degree greater than three. vertices of degree 3 are called Steiner points if they are located in the interior of P . The edges incident to Steiner points make 120° with each other. They are called degenerate Steiner points if they are located on the boundary of P. Degenerate Steiner points can only occur at wide reflex vertices of P. The reader is referred to [17] for basic definitions and properties of ESMTs.

2.1 Polygon Reductions

Consider a unique polygon P' inside P containing the terminals Z, and such that its perimeter is as short as possible. Provan [18] proved that there always exists an ESMT for Z in P completely inP'. Toussaint [19] gave an $O(n(\log n + \log k) + k)$ algorithm to determine P'. The complexity of this algorithm reduces to $O(k)$ if n is fixed. P' is sometimes referred to as the geodesic convex hull for its polygon and its terminals.

3. Construction of the Steiner tree for three points

Torricelli came up with a solution for three points in 1640 [14]. In this solution, the three points are labeled A, B, and C. If we connect them, we have a triangle. If we build three equilateral triangles outside of the ABC triangle, each of which has AB, AC, and BC as one of its sides and we inscribe each of these triangles within a circle, we will have Figure 1.

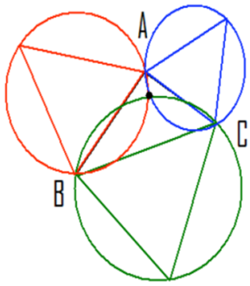

Figure 1. Torricelli's solution for three points

The intersection point of these three circles is the Steiner point we seek which is called the Torricelli point [15]. In 1750, Simpson presented another solution in order to find the Torricelli point [15]. Similar to the previous method, the equilateral triangles outside of the ABC triangle are built. Afterward, the Simpson line is drawn between the vertices of the equilateral triangles. The intersection point of these three lines is the Torricelli point [14]. See Figure 2.

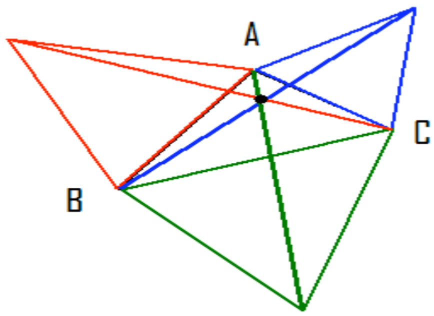

Figure 2. Simpson's solution for three points

The Steiner point is obtained from the intersection of three edges in Figure 2 which are at an angle of 120 degrees with each other. This condition is called the angular condition of the Steiner point (Figure 3).

ISSN : 2322-5157
www.ACSIJ.org

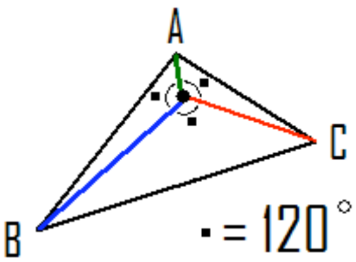

Figure 3. The edges connected to the Steiner point are at an angle of 120 degrees with one another.

4. The Proposed Algorithm Steps

A simple polygon with m vertices, n terminals is presented as an input, as shown in Figure 4.

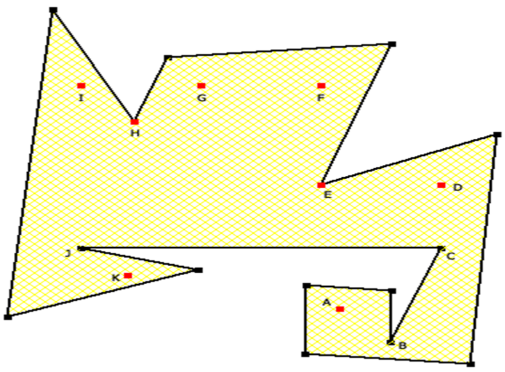

Figure 4. A simple polygon m vertices, n terminals

Our proposed algorithm has three steps.

Step1. First the geodesic convex hull of the terminals in a simple polygon is obtained, which is shown in Figure 5.

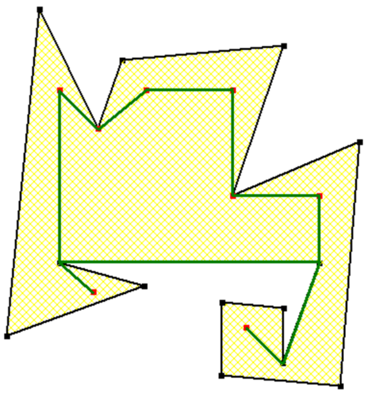

Figure 5. Geodesic convex hull of the terminals in a simple polygon

Step2. The set of terminals and vertices of the geodesic convex hull of Figure 5 are triangulated using the triangulation algorithm [16]. In each triangle constructed whose angles are less than 120 degrees, we obtain the Steiner point using Torricelli or Simpson method (Figure 6).

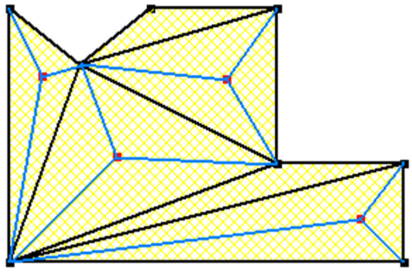

Figure 6. The Steiner point obtained in each triangle

Step3. The result of the second stage of Steiner tree in graph problem. We obtain the Steiner tree on the graph using the algorithm of Milan et al. [9] (Figure 7).

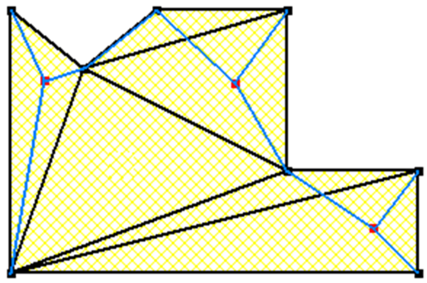

Figure 7. Euclidean Steiner Minimal Tree in the geodesic convex hull

Then the AB, BC and KJ edges are connected to the Steiner tree obtained from the previous stage graph and the resulting tree is called Euclidean Steiner minimal tree.

5. Computational Results

We implemented the proposed algorithm in Delphi programming language and performed our experiments with examples of Soukup [13]. In Table 1, a number of the implemented results are compared with optimum results demonstrating the fact that the proposed algorithm has presented acceptable results.

TABLE 1: PROPOSED ALGORITHM COMPARED WITH SOUKUP EXAMPLES

Example number	***Optimal result***	***Our proposed algorithm***
EX.1	166.44	167.44
EX. 3	159.88	166.53
EX. 9	116.68	117.90
EX. 10	164.28	165.80
EX. 11	381.76	384.96
EX. 19A	223.22	229.22
EX. 19B	281.42	285.71

6. Conclusion

In this paper, the proposed algorithm is able to solve the Steiner tree problem within a simple polygon on the Euclidean plane. The calculation results of the mentioned algorithm are easy in terms of implementation and they lead to acceptable results.

References

[1] M. R. Garey, R.L.Graham, D.S. Johnson, “The complexity of computing Steiner minimal trees”, SIAM J. Appl. Math, 1977

[2] D.M. Warme, P. Winter, M. Zachariasen, “Exact algorithms for plane Steiner tree problems: a computational study”, in: D.-Z. Du,J.M. Smith, J.H. Rubinstein (Eds.), Advances in Steiner Trees, Kluwer, Dordrecht, 2000, pp. 81–116.

[3] J.E. Beasley, “A heuristic for the Euclidean and rectilinear Steiner tree problem”, E uropean J. Oper. Res. 58, 1992, 284–292.

[4] J. MacGregor Smith, D.T. Lee, J.S. Liebman, “An an $O(n\log n)$ heuristic for the Steiner minimal tree problem on the Euclideanmetric”, Networks 11, 1981, 23–29.

[5] Boyce, W.M., and Seery, J.B., “STEINER72, an improved version of Cockayne and Schiller's program ”, Computer Science Technical Report Number 35, Bell Laboratories, Murry Hill, NJ, 1972.

[6] Cockayne, E.J., and Schiller, D.G., “Computation of Steiner minimal trees”, in: D.J.A Welsh and D. R. Wood (eds.),Combinatorics, Institute of Mathematics and its Application,Southend, 1972, pp. 53–71.

[7] Winter, P., “An algorithm for the Steiner problem in the Euclidean plane”, Networks 15, 1985, pp. 323–345.

[8] Cockayne, E.J., and Hewgill, D.E., “Exact computation of Steiner minimal trees in the plane”, Information Processing Letters 22,1986, pp. 151–156.

[9] Chang, S.K., “The generation of minimal trees with a Steiner topology”, Journal of the ACM 19, 1972, pp.699–711.

[10] Korhonen, P., “An algorithm for transforming a spanning tree into a Steiner tree”, Survey of Mathematical Programming (Proceedings of the 9th International Mathematical Programming Symposium) 2, 1980, pp.349–357.

[11] Smith, J.M., and Leibman, J.S., “Steiner trees, Steiner circuits and the interference problem in building design”, Engineering Optimization 4, 1980, pp. 15–36.

[12] Lundy, M., “Applications of the annealing algorithm to combinatorial problems in statistics”, Biometrika 72, 1985, pp.191–198.

[13] J. Soukup, W.F. Chow, “set of test problems for the minimum length connection networks”, ACM SIGMAP Bulletin, 1973.

[14] G. Ausiello, P. Crescenzi, G. Gambosi, V. kann, A. Marchetti Spaccamela and M. Protasi, Complexity and Approximation, Springer Verlag, 1999.

[15] A. O. Ivanov and A. A. Tuzhilim, "Minimal Networks: The Steiner Problem and its Generalizations," CRC Press, 1994.

[16] S. Fortune, A sweepline algorithm for Voronoi diagrams. Algorithmica, 2(2):pp.153-174, 1987.

[17] F.K.Hwang, D.S. Richards and P. Winter, The Steiner Tree Problem, North-Holland(1992).

[18] J.S. Provan, An apporximation scheme for finding Steiner trees with obstacles,SIAM J.Comput.17(1988)920-934.

[19] G.T. Toussaint, Computing geodesic properties inside a simple polygon, Revue D'intelligence Artificiell 3(1989)9-12.