Lossy Color Image Compression Based on Singular Value Decomposition and GNU GZIP

Jila-Ayubi [1], Mehdi-Rezaei [2]

[1] Department of Electrical engineering, Meraaj Institue, Salmas , Iran
jila.ayubi@gmail.com
[2] Faculty of Electrical and Computer Engineering, University of Sistan and Baluchestan, Zahedan, Iran
mehdi.rezaei@ece.usb.ac.ir

Abstract

In matrix algebra, the Singular value decomposition (SVD) is an factorization of complex matrix that has been applied to principal component analysis, canonical correlation in statistics, the determination of the low rank approximation of matrices. In this paper, using the SVD and the theory of low rank approximation of a matrix, we offer a new scheme for color image compression based on singular value decomposition and gzip. The scheme focuses on color images, thus fitting various network multimedia applications. SVD is applied to color image for low rank approximation. This compression scheme may have applications in sound and video compression.GNU zip is a compression utility designed to be a replacement for compress. Its main advantages over compress are much better compression and freedom from patented algorithms. The aim is to improve a fast procedure of computation and simple implementation of the algorithm. The performance of the new compression based on SVD and GNU GZIP is examined.

Keywords: Singular Value Decomposition, Image compression, low rank approximation, GNU GZIP.

1. Introduction

Image compression is an important aspect of digital image processing [1]. It is used, for instance, for image transmission, like television, and image storage, like fingerprints. Current research in this field is very active.

Data compression, in general, is either lossless (original data can be totally recovered after decompression) [2] or lossy (data compression techniques in which some amount of the original data is lost). Lossy data compression has received significant attention from the research community due to its potential to achieve higher compression ratio (CR). In addition, in compressing image data the non-linearity of the human visual system can be used as basis
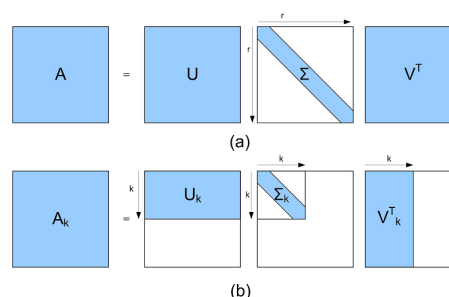

Fig. 1 Singular value decomposition of A (a) full-rank r (b) rank-k approximation.

for striking a compromise (to a certain extent) between the image's perceptual quality and the desire to achieve high CR performance. File compression and decompression time requirements are not insignificant. Intuitively, the algorithms achieving the best compaction are usually not the fastest; accordingly, choices must be made for each circumstance. Some compression programs offer users the choice of lossless or lossy, considering the decision between speed versus compression ratio. Ultimately, lossy algorithms are usually the method of choice when regarding the compression of image data [3]. Singular Value Decomposition (SVD) is said to be a significant topic in linear algebra by many renowned mathematicians. SVD was introduced by Eckart and Young [4] and has become one of the most widely used techniques of computational algebra and multivariate statistical analysis applied for data approximation, reduction and visualization. The SVD, is also known in terms of matrix spectral decomposition, is closely related to principal components and Moore Penrose generalized matrix inverse.

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Old Input Data   Sliding Window   New Input Data

...KNMDVQER..YMPIQUETRCEBBGZEBZTQMGHE...

Fig. 2 Sliding window for the search of repetitions.

SVD presents a rectangular matrix via a low rank additive combination of the outer products of dual right and left eigenvectors [5], [6], [7], [8].The use of singular value decomposition (SVD) in image compression has been widely studied [9], [10], [11], [12].

The rest of the Letter is organized as follows. Section II describes fundamental of singular value decomposition. In Section III, compression and decompression process is proposed. Also, the selected example and simulation results are discussed in Section IV. Section V is the conclusion.

2. Singular Value Decomposition

An m n matrix A can be factorized as

$$A = U\Sigma V^T \tag{1}$$

or

$$A = \sum u_i \sigma_i v_i = u_1 \sigma_1 v_1^T + u_2 \sigma_2 v_2^T + ... + u_r \sigma_r v_r^T \tag{2}$$

where U is an $m \times m$ orthogonal matrix, V is an $n \times n$ orthogonal matrix, Σ is an $m \times n$ diagonal matrix with non-negative entries as follows

$$\Sigma_{m \times n} = \begin{bmatrix} D & O_1 \\ O_2 & O_3 \end{bmatrix} \tag{3}$$

where O_1, O_2, O_3 are zero matrices and D is a diagonal matrix whose diagonal entries Σ have nonzero singular values of A

$$D = \begin{bmatrix} \sigma_1 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 \\ 0 & 0 & . & 0 \\ 0 & 0 & 0 & \sigma_r \end{bmatrix}, \sigma_1 \geq \sigma_2 \geq ... \geq \sigma_r \geq 0 \tag{4}$$

where r is the rank of A.

The factorization in (1) is called the singular value decomposition of A.For a matrix with more rows than columns, in an alternate definition of the singular value decomposition, the matrix U is $m \times n$ with orthogonal columns, and Σ is an $m \times m$ diagonal matrix with no-negative entries. Likewise, for a matrix with more columns than rows, the singular value decomposition can be defined above but with the matrix V being $n \times m$ with orthogonal columns, and Σ is an $m \times m$ and diagonal with no-negative entries.

Data Shifting

Output

Shifting window	Input Data	Literal	Match
	A B C D E A C D E A C D	A	-
	A B C D E A C D E A C D	B	-
	A B C D E A C D E A C D	C	-
	A B C D E A C D E A C D	D	-
	A B C D E A C D E A C D	E	-
	A B C D E A C D E A C D	A	-
	A B C D E A C D E A C D	-	4 , 2
	A B C D E A C D E A C D	-	4 , 2

Fig. 3 LZ77 longest match searching.

Given an $m \times n$ matrix A, a rank-k approximation of A is a matrix A_k of the same size and of rank at most k that minimizes that difference with A.A rank-k approximation to A is obtained by taking the first k terms of the SVD

$$A_k = \sum_{i=1}^{k} u_i \sigma_i v_i^T \tag{5}$$

In general, low-rank approximations of data matrices serve two proposes: they reduce space requirements and often provide a more transparent representation. Fig.1 show singular value decomposition and low-rank approximations of matrix A.

3. GNU GZIP

GZIP is a lossless compression standard, since after decompression all information will be reconstructed identical to the original data. The compression algorithm [13] itself is defined as a combination of the Lempel-Ziv 77 (LZ77) algorithm and an adopted Huffman encoding. The LZ77 [14] is a dictionary based algorithm. A sliding window of fixed size allocates a backward buffer and all input data is compared to substrings within this dictionary. This compression algorithm works sequentially and one basic operation is the sequential shifting of new input through this window, as illustrated in Fig.2 from the right to the left side.

"Old" data is shifted out while"new" data is shifted in. Recurrence searching is started by reading a byte from the current input data. Cells within the sliding buffer may be found to match the data. All matched cells are marked and and the next input byte is read. Again matches to the second second byte may be found within this buffer and if these cells are connected to further marked cells, they show

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recurrences with an overall length of two. Other matches, not connected to the first ones, are shorter than the other ones and can be dropped. This process continues sequentially until there is no related longer match any more. If the longest match has just a length of one or two bytes, the bytes itself is the output. Otherwise, in the case of a longer match, the corresponding input sequence will be replaced by a table containing information about the backward distance and the overall match length. An example LZ77 operation is shown in Fig.3 to clarify the searching procedure.

The output produced by LZ77 will be either a literal or a pair of distance and length information. Next, these outputs are encoded by a Huffman encoder [15].It uses a statistical approach, whose essential idea is to encode sequences of high probability by means of short bit strings while long bit strings describe sequences of low probability. Definitions of all replacements for literals, distances and lengths are looked up within an encoder tree. Various conflagrations for the LZ77 and Huffman algorithms can be selected. First, the LZ77 sliding window size may contain up to 32 Kbytes. Further more, the Huffman encoder can operate in dynamic or static modes. Thus, a specific header needs to be attached to the data file in order to describe the configuration. Finally, a footer with an Adler32 checksum for error detection is attached which completes the file.

4. COMPRESSION AND DECOMPRESSION PROCESS

In this paper, color images are used for compression. Color is important visual information which keeps humans fascinated since birth. The representation of color is based on the classical three-color theory where any color can be reproduced by mixing an appropriate set of three primary colors. Color information is commonly represented in the widely used RGB (red, green, blue) Cartesian coordinate system. An RGB color image, represented by 8 bits of R, G, and B pixels has 256^3 or 16,777, 216 colors.

The compression and decompression process are as follows.

4.1 Image Compression process

In this section the algorithm of our proposed scheme are discussed. The image compression algorithm proposed in this paper consists of the following major parts:

- Color image is divided into three channels. Each channel has an independent process, and finally will be combined.

- Singular value Decomposition for each channel separately is calculated by (2). U_R, Σ_R and V_R are singular value for Red channel. Similarly, U_G, Σ_G and V_G are singular value of Green channel and U_B, Σ_B and V_B are singular value of Blue channel.
- In the next step best rank for each channel will be selected. The rank of r should be selected so that the original image with the estimated image error is less.
- Finally, matrices obtained from each channel can be compressed with gzip algorithm and the output file is stored.

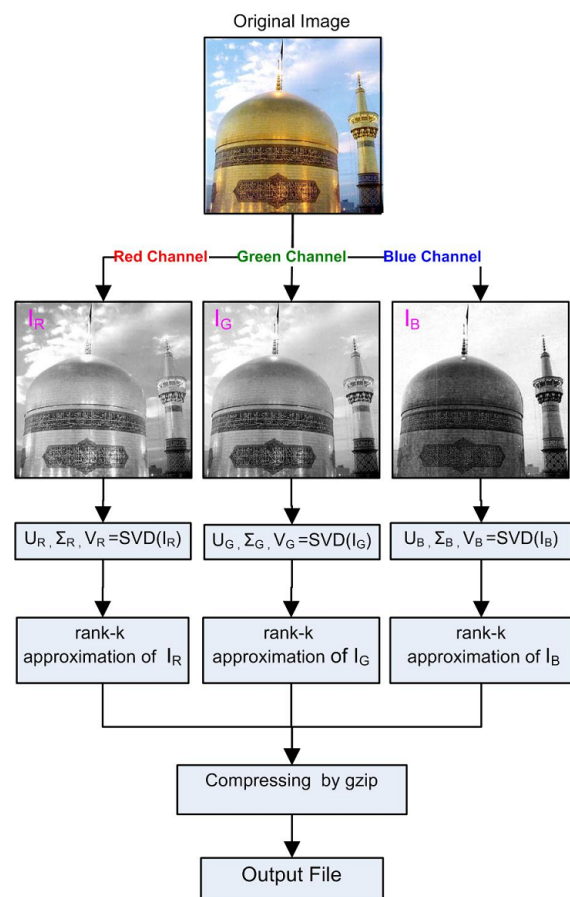Flowchart of compression process shown in Fig.4.

Fig. 4 Flowchart of Compression process.

4.2 Image decompression process

The image decompression process is very similar to image compression process. The image decompression process consists of the following major parts:

- Firstly, the gzip input image must be decompressed

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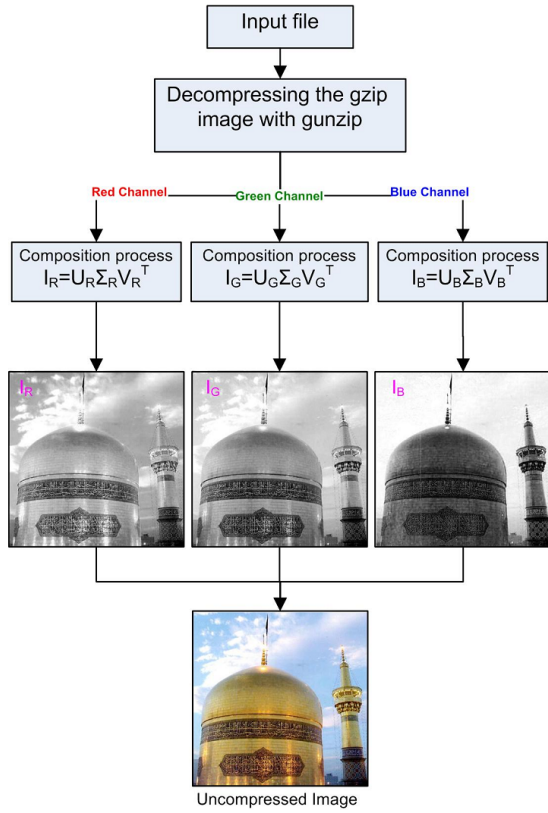www.ACSIJ.org

with gunzip.

Fig. 5 Flowchart of Decompression process.

- After gunzip decompression, matrices for each channel can be obtained from input file.
- Singular value of each channel is reconstructed red, green and blue channels.
- The channels obtained from previous step are combined together for output image construction.

Flowchart of decompression process shown in Fig.5.

5. NUMERICAL RESULT

In this section the experimental results of our proposed scheme are discussed. To demonstrate the efficiency of the proposed algorithm, MATLAB simulations are performed by using 512×512 pixel RGB color Mashhad image. Fig.5 demonstrates the proposed algorithm.

Fig. 6 (a-f) are the above experiment, when a 512×512 Mashhad image is reconstructed by 1, 5, 10, 50, 100, 512 singular values respectively.

To quantitatively evaluate the performance of the proposed scheme, the peak Signal to Noise Ratio (PSNR) for an original $m \times n \times k$ image, X, and the reconstructed image, $\hat{x}$, are defined as follows:
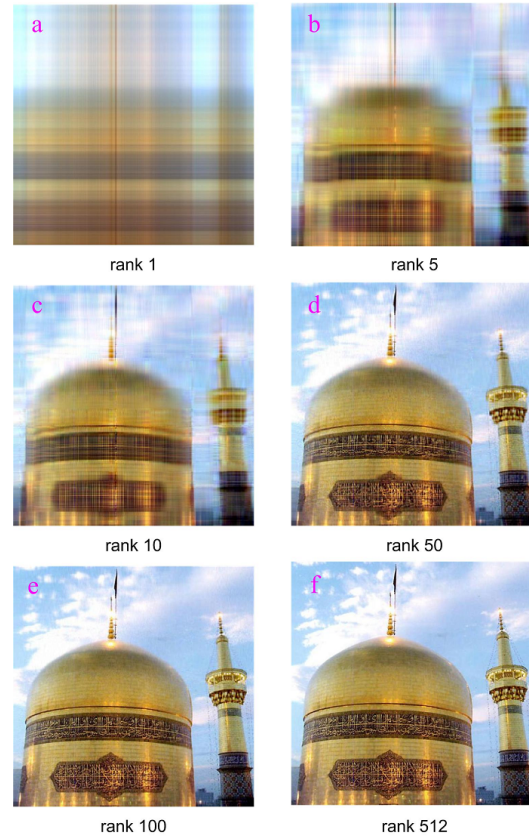

Fig. 6 (a-f) MASHHAD image reconstruction by 1, 5, 10, 50, 100, 512 singular values respectively

$$PSNR = 10 \log_{10} \left(\frac{255^2 mnk}{\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{k} [X(i,j,l) - \hat{x}(i,j,l)]} \right) \quad (6)$$

$$= 10 \log_{10} \left(\frac{255^2 mnk}{\| X - \hat{x} \|_F^2} \right)$$

In addition to PSNR, compression ratio (CR) is required to compare compression performance. CR is given by :

$$CR = \frac{N_O}{N_C} \quad (7)$$

where N_O , N_C are number of bits in original and compressed file respectively.

Also, common color images in papers is used for PSNR and CR simulation.Fig.7 show four samples of common images in image processing papers. TABLE I illustrates result of calculated PSNR in common images with different rank of approximation. TABLE II is similar to the previous table and just CR is simulated with same ranks and same images.

Fig. 7 Tested images (a)Baboon (b)Fruits (c)Peppers (d)Monarch.

6. Conclusion

We propose a new scheme for image compression based on singular value decomposition and GZIP. The scheme focuses on color images, thus fitting various network multimedia applications. SVD is applied to color image for low rank approximation. We have used best low rank to increase compression ratio(CR) and performance involved in the algorithm. We have presented a fast procedure of computation and simple implementation of the algorithm. Therefore, may be an effective technique for color image compression. This compression scheme may have applications in sound and video compression. The aim of this work was to realize a compression method with lossy data. So further studies must be started to develop compression methods with lossless data.

TABLE 1: SIMULATION RESULT OF PSNR IN DIFFERENT TESTED IMAGES

Rank	Mashhad	Baboon	Fruits	Peppers	Monarch
1	65.10	64.41	66.68	64.31	66.68
5	69.92	66.01	68.65	67.07	68.65
10	72.39	66.08	70.54	68.92	70.54
50	79.01	68.51	78.77	79.36	78.77
100	87.46	71.08	86.18	86.10	86.18

TABLE 2: SIMULATION RESULT OF CR IN DIFFERENT IMAGES

Rank	CR in all images(512×512)
1	64.00
5	12.80
10	6.40
50	1.40
100	0.71

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Jila ayubi obtained her BSc and degrees in electrical engineering from University of Orumieh, Iran in 2009 and she earned her MSc degree from the University of Sistan and Baluchestan, Iran in 2011. Her areas of research include signal and image processing.

Mehdi Rezaei received his BSc and MSc degrees in electrical engineering from Amir Kabir University of Technology, Tehran and Tarbiat Modares University of Tehran, Iran in 1992 and 1996, respectively. In 1996, he joined the University of Sistan and Baluchestan, Iran. He earned his PHD degree in communication engineering from the Tampere University of Technology, Finland in 2008. As an assistant professor at the University of Sistan and Baluchestan, his research interests are signal and image processing and video coding. He published several Journal and conference papers in field of image and video processing.

Dr Rezaei got the Nokia Foundation Award in 2005 and 2006.